

# Fair and Square

What can we learn from Magic, Latin  
and Vedic number squares?



# Emperor Yu and the Turtle



# Latin Squares made from 4, 5 and 6

|   |   |   |
|---|---|---|
| 4 | 6 | 5 |
| 6 | 5 | 4 |
| 5 | 4 | 6 |

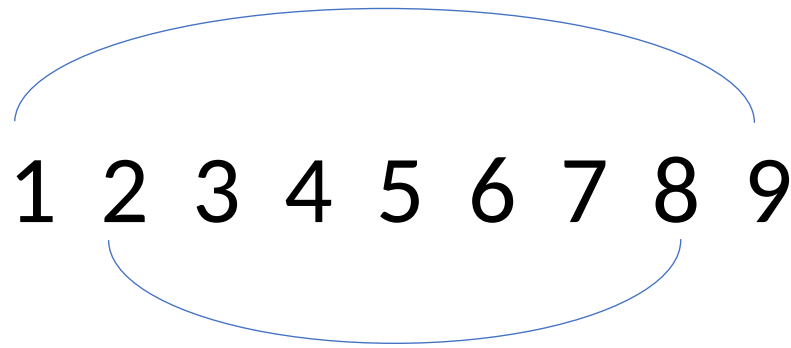
|   |   |   |
|---|---|---|
| 5 | 6 | 4 |
| 4 | 5 | 6 |
| 6 | 4 | 5 |

|   |   |   |
|---|---|---|
| 5 | 4 | 6 |
| 6 | 5 | 4 |
| 4 | 6 | 5 |

What patterns do you notice?

Can anymore different squares be made with these numbers?

**Can you find a quick way of adding all these numbers together?**



total = ?

# The Luo Shu Magic Square

|   |   |   |
|---|---|---|
| 4 | 9 | 2 |
| 3 | 5 | 7 |
| 8 | 1 | 6 |

|   |   |   |
|---|---|---|
| 四 | 九 | 二 |
| 三 | 五 | 七 |
| 八 | 一 | 六 |

**How many ways can you arrange the numbers so that the rows, columns and diagonals still add up to 15?**

(15 is known as the 'magic constant'.)

|   |   |   |
|---|---|---|
| 4 | 9 | 2 |
| 3 | 5 | 7 |
| 8 | 1 | 6 |

|   |   |   |
|---|---|---|
| 8 | 3 | 4 |
| 1 | 5 | 9 |
| 6 | 7 | 2 |

|   |   |   |
|---|---|---|
| 6 | 1 | 8 |
| 7 | 5 | 3 |
| 2 | 9 | 4 |

|   |   |   |
|---|---|---|
| 2 | 7 | 6 |
| 9 | 5 | 1 |
| 4 | 3 | 8 |

|   |   |   |
|---|---|---|
| 8 | 1 | 6 |
| 3 | 5 | 7 |
| 4 | 9 | 2 |

|   |   |   |
|---|---|---|
| 6 | 7 | 2 |
| 1 | 5 | 9 |
| 8 | 3 | 4 |

|   |   |   |
|---|---|---|
| 2 | 9 | 4 |
| 7 | 5 | 3 |
| 6 | 1 | 8 |

|   |   |   |
|---|---|---|
| 4 | 3 | 8 |
| 9 | 5 | 1 |
| 2 | 7 | 6 |

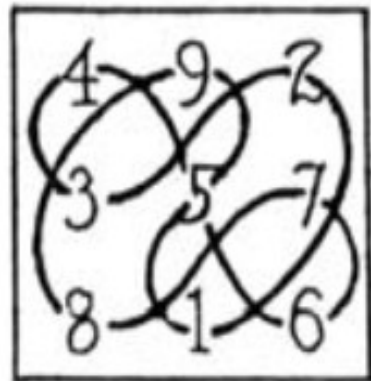
What do you notice about these variations?

# Luo Shu Magic Square with 5 taken from each number

|    |    |    |
|----|----|----|
| -1 | +4 | -3 |
| -2 | 0  | +2 |
| +3 | -4 | +1 |

What do you notice about this pattern?

# What patterns can be found in the Luo Shu Square?



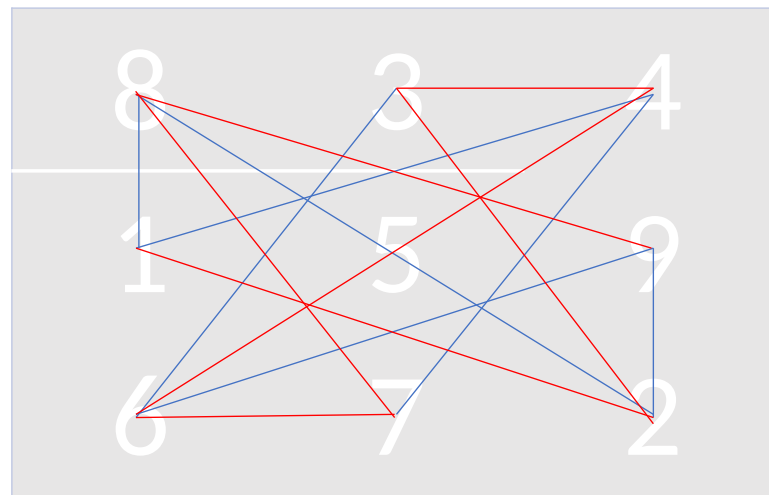
THE 3 X 3  
SQUARE



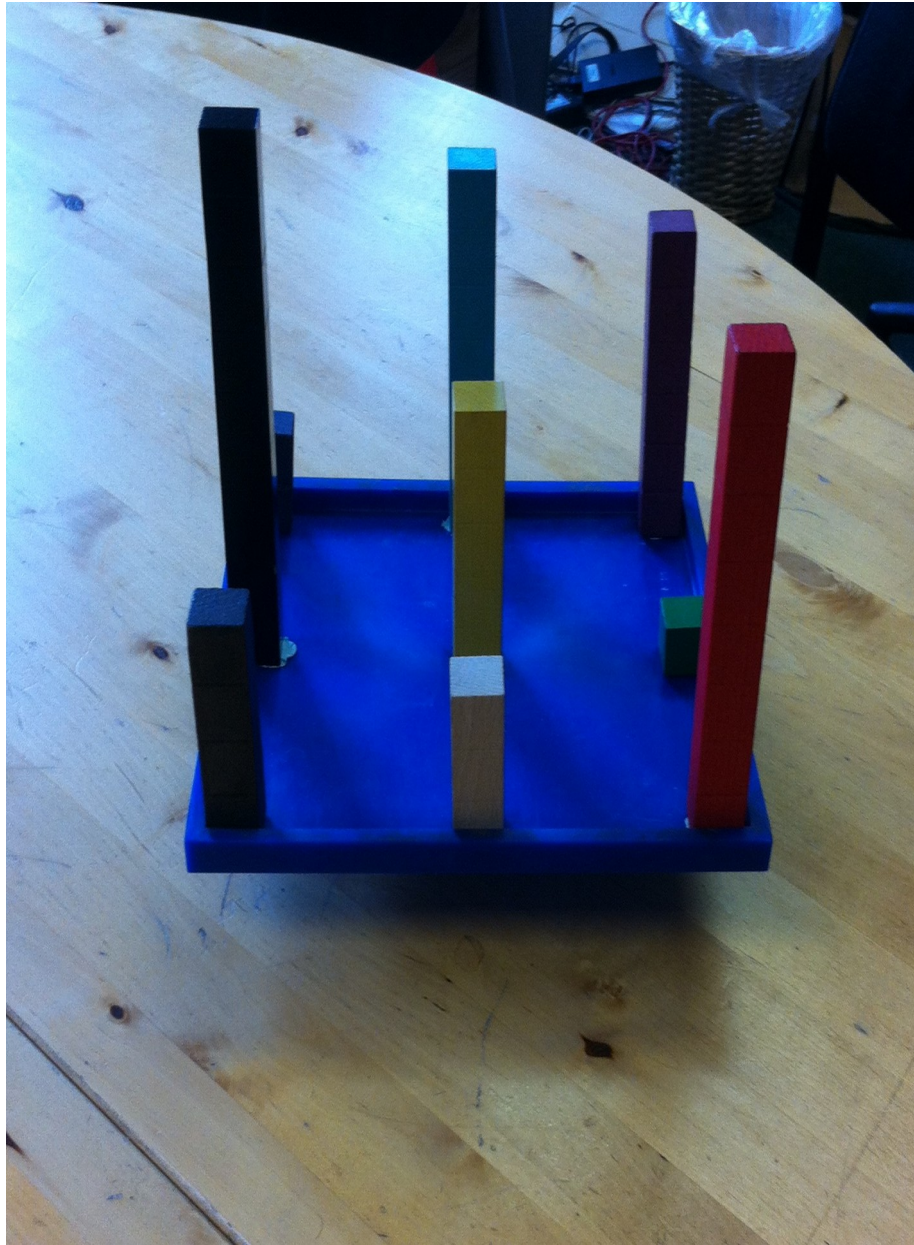
DESIGN FOR A VENTILATING REGISTER  
DERIVED FROM THE  
MAGIC LINE OF THE  
MAGIC SQUARE OF 3

figure 1. Claude Bragdon,  
*Frozen Fountain*, 1932

# What patterns can be found in the Luo Shu Square?

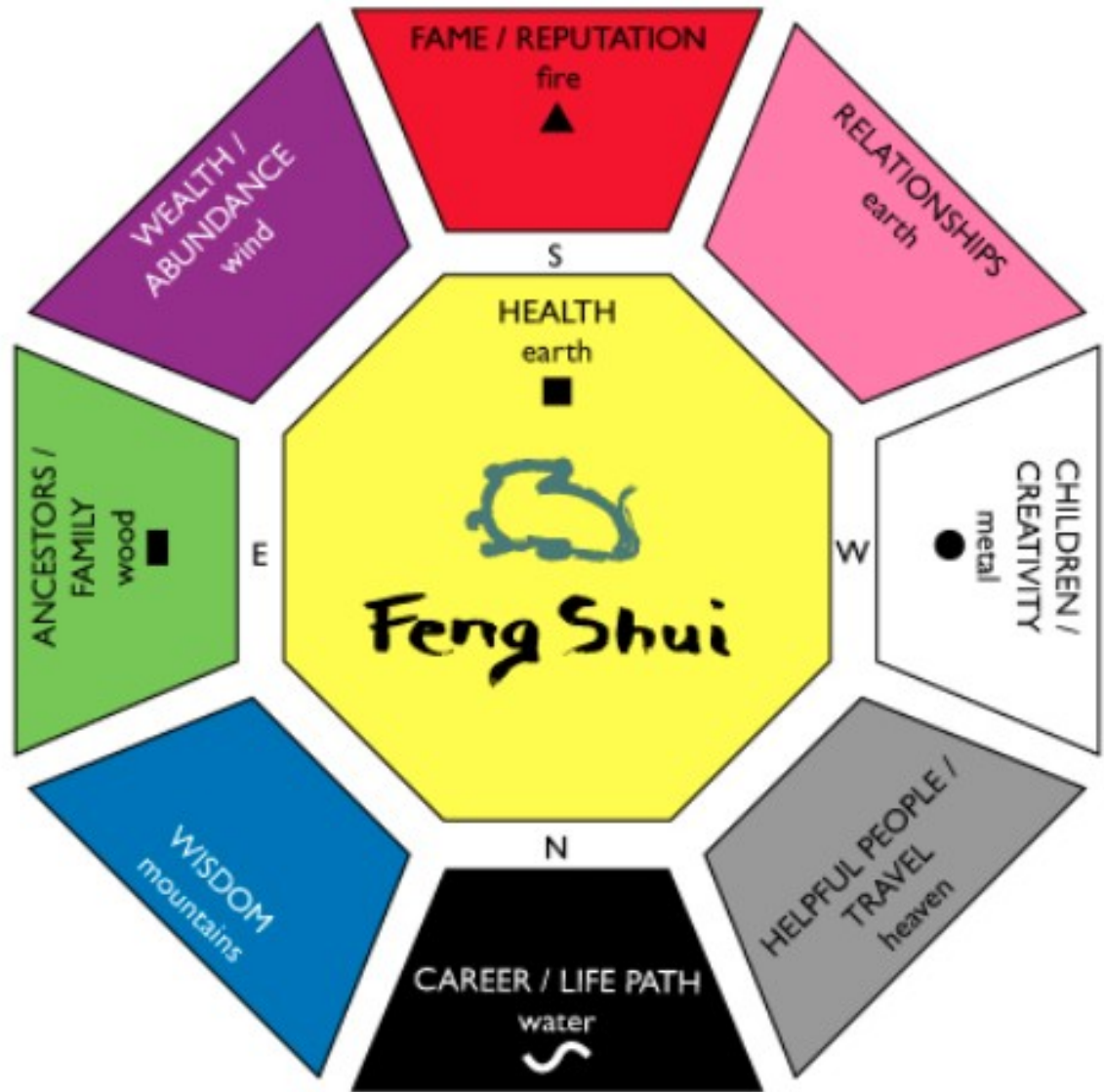


**Balancing  
a magic  
square,  
can it be  
done?**



# Feng Shui Bagua

Turn the paper until 'fire' is pointing south. Look at the Bagua octagon and the classroom. Do you notice any correspondences? The bagua is also about balancing 8 different areas of life.



The one thousand year old Jaina square from Parshvanatha temple in Madhya Pradesh in India. Do you recognise any of the numbers?

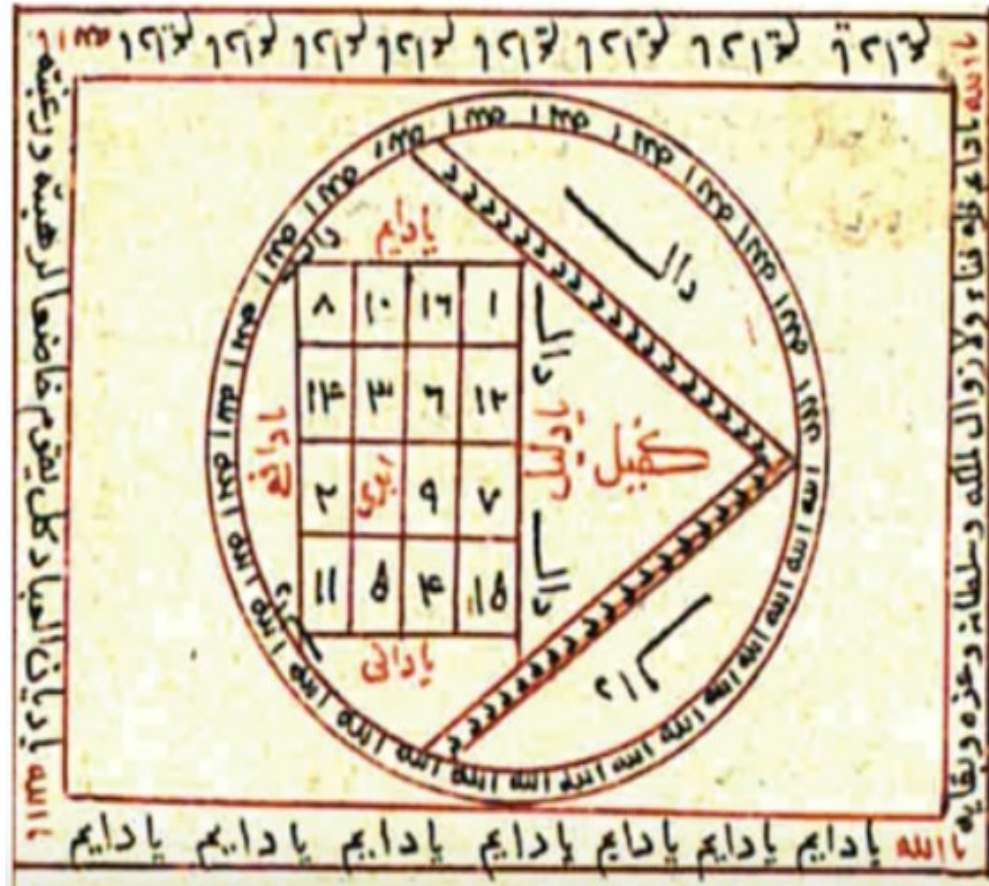


|    |    |    |    |
|----|----|----|----|
| 7  | 12 | 1  | 14 |
| 2  | 13 | 8  | 11 |
| 16 | 3  | 10 | 5  |
| 9  | 6  | 15 | 4  |

What is the magic constant? In how many ways can you reach it?

**An Islamic  
magic  
square  
from the  
*Shams al-  
Ma'arif* by  
Ahmed al-  
Buni  
1225CE**

Do you  
recognise  
any of the  
numbers in  
the  
square?



|    |    |    |    |
|----|----|----|----|
| 8  | 11 | 15 | 1  |
| 14 | 2  | 7  | 12 |
| 3  | 17 | 9  | 6  |
| 10 | 5  | 4  | 16 |

What is the magic constant? (Add up any one of the straight lines of numbers to find this.) In how many ways can you reach it?

# What numerals were being used in Europe at this time?



And even more recently. Can anyone work out what year this is? (M=1000, D=500, C=100, X=10, V=5, I=1)



# How the numbers we use reached Europe.

Fibonacci, from Pisa (which is now in Italy) spent time in North Africa where he learnt about how effective the system of numbers used by the Arabs was. The numbers originally came from India. He published a book in 1202CE and the numbers gradually got

— = ≡ ≠ √ √ | 6 7 8 9

Brāhmī



१ २ ३ ४ ५ | ६ ७ ८ ९ ०

Indien (Gwâlîor)



१ २ ३ ४ ५ | ६ ७ ८ ९ ०

Sanskrit-Devanāgarī (Indien)



١ ٢ ٣ ٤ ٥ | ٦ ٧ ٨ ٩

Arabe occidental (Gobâr)



١ ٢ ٣ ٤ ٥ | ٦ ٧ ٨ ٩ ٠

Arabe oriental

# Srinivasa Ramanujan

## (1887 – 1920)



An Indian mathematical genius who although had almost no training in mathematics, found solutions to problems considered to be unsolvable. He often said '*An equation for me has no meaning unless it represents a thought of God*'. He was invited to come to Cambridge by the mathematician Prof. Godfrey Hardy.

Hardy remembers once going to see him when he was ill in hospital. '*I had ridden in taxi cab number 1729 and remarked that the number seemed to me rather a dull one, and that I hoped it was not an unfavorable omen. "No", he replied, "it is a very interesting number; it is the smallest number expressible as the sum of two cubes in two different ways."*'

Every positive integer was said to be one of his personal friends. His magic square appears on the next slide.

DD

MM

CC

YY

YY+1

CC-1

MM-3

DD+3

MM-2

DD+2

YY+2

CC-2

CC+1

YY-1

DD+1

MM-1

| DD | MM | CC | YY |
|----|----|----|----|
| 22 | 12 | 18 | 87 |
| 88 | 17 | 9  | 25 |
| 10 | 24 | 89 | 16 |
| 19 | 86 | 23 | 11 |

| DD         | MM         | CC         | YY         |            |
|------------|------------|------------|------------|------------|
| 22         | 12         | 18         | 87         | <b>139</b> |
| 88         | 17         | 9          | 25         | <b>139</b> |
| 10         | 24         | 89         | 16         | <b>139</b> |
| 19         | 86         | 23         | 11         | <b>139</b> |
| <b>139</b> | <b>139</b> | <b>139</b> | <b>139</b> | <b>139</b> |

How many other ways can you find to reach 139 in Ramanujan's magic square?

|    |    |    |    |
|----|----|----|----|
| 22 | 12 | 18 | 87 |
| 88 | 17 | 9  | 25 |
| 10 | 24 | 89 | 16 |
| 19 | 86 | 23 | 11 |

|    |    |    |    |
|----|----|----|----|
| 22 | 12 | 18 | 87 |
| 88 | 17 | 9  | 25 |
| 10 | 24 | 89 | 16 |
| 19 | 86 | 23 | 11 |

|    |    |    |    |
|----|----|----|----|
| 22 | 12 | 18 | 87 |
| 88 | 17 | 9  | 25 |
| 10 | 24 | 89 | 16 |
| 19 | 86 | 23 | 11 |

|    |    |    |    |
|----|----|----|----|
| 22 | 12 | 18 | 87 |
| 88 | 17 | 9  | 25 |
| 10 | 24 | 89 | 16 |
| 19 | 86 | 23 | 11 |

|  |  |  |  |

|    |    |    |    |
|----|----|----|----|
| 22 | 12 | 18 | 87 |
| 88 | 17 | 9  | 25 |
| 10 | 24 | 89 | 16 |
| 19 | 86 | 23 | 11 |

|    |    |    |    |
|----|----|----|----|
| 22 | 12 | 18 | 87 |
| 88 | 17 | 9  | 25 |
| 10 | 24 | 89 | 16 |
| 19 | 86 | 23 | 11 |

|    |    |    |    |
|----|----|----|----|
| 22 | 12 | 18 | 87 |
| 88 | 17 | 9  | 25 |
| 10 | 24 | 89 | 16 |
| 19 | 86 | 23 | 11 |

|    |    |    |    |
|----|----|----|----|
| 22 | 12 | 18 | 87 |
| 88 | 17 | 9  | 25 |
| 10 | 24 | 89 | 16 |
| 19 | 86 | 23 | 11 |

What do you notice about the pattern of the words in this Roman inscription? Does this follow the rules of a magic number square?



# Latin Square

|   |   |   |
|---|---|---|
| 1 | 2 | 3 |
| 3 | 1 | 2 |
| 2 | 3 | 1 |

You can make puzzles from Latin squares called Sudokus. A good Sudoku is one that has a unique solution.

|   |  |   |
|---|--|---|
| 1 |  |   |
|   |  | 3 |
|   |  |   |

|   |   |  |
|---|---|--|
| 1 | 3 |  |
|   |   |  |
|   |   |  |

|   |   |  |
|---|---|--|
| 1 |   |  |
|   | 3 |  |
|   |   |  |

Two of these are good Sudokus but one has more than one (apparent) solution.

*Which one?*

*Mathematicians would probably call all these solutions the same.*

*Why?*

*Do you agree?*

**Look at your school timetable. Notice how it works like a Latin Square (with each teacher and class appearing only once in each time slot on a given day)**

| WHOLE SCHOOL TIMETABLE – Autumn 2017 |                |                                      |                                |                       |                      |                                       |                       |                                            |                      |                                 |                      |                                 |                      |                                      |                      |                                            |                      |                                         |                       |                      |                      |
|--------------------------------------|----------------|--------------------------------------|--------------------------------|-----------------------|----------------------|---------------------------------------|-----------------------|--------------------------------------------|----------------------|---------------------------------|----------------------|---------------------------------|----------------------|--------------------------------------|----------------------|--------------------------------------------|----------------------|-----------------------------------------|-----------------------|----------------------|----------------------|
| Time                                 |                | Monday                               |                                |                       |                      | Tuesday                               |                       |                                            |                      | Wednesday                       |                      |                                 |                      | Thursday                             |                      |                                            |                      | Friday                                  |                       |                      |                      |
|                                      |                | K53                                  | Yr10                           | 11a                   | 11b                  | K53                                   | Yr10                  | 11a                                        | 11b                  | K53                             | Yr10                 | 11a                             | 11b                  | K53                                  | Yr10                 | 11a                                        | 11b                  | K53                                     | Yr10                  | 11a                  | 11b                  |
| 9.00-9.30                            |                | Breakfast Club (NC +TA)              |                                |                       |                      | Breakfast Club (NC +TA)               |                       |                                            |                      | Breakfast Club (NC +TA)         |                      |                                 |                      | Breakfast Club (NC +TA)              |                      |                                            |                      | Breakfast Club (NC +TA)                 |                       |                      |                      |
| Reg                                  | 9.30-9.45      | TUTORIAL                             |                                |                       |                      | TUTORIAL                              |                       |                                            |                      | TUTORIAL                        |                      |                                 |                      | TUTORIAL                             |                      |                                            |                      | TUTORIAL                                |                       |                      |                      |
| P1                                   | 09.45 to 10.30 | REP SS (Lib)                         | ENGLISH COG (Eng Rm)           | SCIENCE FOS (Sci Rm)  | MATHS DT (Ma Rm)     | ART RSA (Ar Rm)                       | MATHS DT (Ma Rm)      | SCIENCE FOS (Sci Rm)                       | ENGLISH COG (Eng Rm) | COMP SC DT (IT Rm)              | SCIENCE FOS (Sci Rm) | Music Brent Music + TAs (Cabin) |                      | SCIENCE FOS (Sci Rm)                 | ENGLISH COG (Eng Rm) | ART RSA (Ar Rm) or COMP SCIENCE DT (IT Rm) |                      | PHISE RSA (Lib)                         | PHISE JE (Eng Rm)     |                      |                      |
| P2                                   | 10.30 to 11.15 | ART RSA (Ar Rm)                      | SCIENCE FOS (Sci Rm)           | MATHS DT (Ma Rm)      | ENGLISH COG (Eng Rm) | HISTORY COG (En Rm)                   | ENGLISH JE (Lib)      | ART RSA (Ar Rm) or COMP SCIENCE DT (IT Rm) |                      | Music Brent Music + TAs (Cabin) |                      | MATHS DT (Ma Rm)                | ENGLISH COG (Eng Rm) | ENGLISH SS (Eng Rm)                  | SCIENCE FOS (Sci Rm) |                                            |                      | FOOD TECH RSA (FD Rm)                   | SCIENCE FOS (Sci Rm)  | ENGLISH COG (Eng Rm) | MATHS DT (Ma Rm)     |
| Break                                |                | Resilience/ Mindfulness(SS, JE +TAs) |                                |                       |                      | Resilience/ Mindfulness(SS, FOS +TAs) |                       |                                            |                      | Assembly (All Staff)            |                      |                                 |                      | Resilience/ Mindfulness(SS, DT +TAs) |                      |                                            |                      | Resilience/ Mindfulness (COG, RSH +TAs) |                       |                      |                      |
| P3                                   | 11.30 to 12.15 | ENGLISH SS (Lib)                     | SCIENCE FOS (Sci Rm)           | HISTORY COG (Eng Rm)  |                      | ENGLISH SS (Lib)                      | HISTORY COG (Eng Rm)  | SCIENCE FOS (Sci Rm)                       | MATHS DT (Ma Rm)     | Music Brent Music + TAs (Cabin) |                      | HISTORY COG (Eng Rm)            |                      | HISTORY COG (En Rm)                  | ART RSA (Ar Rm)      | SCIENCE FOS (Sci Rm)                       | MATHS DT (Ma Rm)     | ENGLISH JE (Lib)                        | HISTORY COG (Eng Rm)  | MATHS DT (Ma Rm)     | SCIENCE FOS (Sci Rm) |
| P4                                   | 12.15 to 13.00 | SCIENCE FOS (Sci Rm)                 | MATHS DT (Ma Rm)               | FOOD TECH RSA (FD Rm) |                      | REP SS (Lib)                          | FOOD TECH RSA (FD Rm) | MATHS DT (Ma Rm)                           | SCIENCE FOS (Sci Rm) | SCIENCE FOS (Sci Rm)            | MATHS DT (Ma Rm)     | FOOD TECH RSA (FD Rm)           |                      | MATHS DT (Ma Rm)                     | DT CD (Lib)          |                                            | ENGLISH COG (Eng Rm) | MATHS DT (Ma Rm)                        | FOOD TECH RSA (FD Rm) | SCIENCE FOS (Sci Rm) | ENGLISH JE (Lib)     |
| Lunch                                |                |                                      |                                |                       |                      |                                       |                       |                                            |                      |                                 |                      |                                 |                      |                                      |                      |                                            |                      |                                         |                       |                      |                      |
| P5                                   | 13.30 to 14.15 | MATHS DT (Ma Rm)                     | ART RSA (Ar Rm) or DT CD (Lib) | ENGLISH COG (Eng Rm)  | SCIENCE FOS (Sci Rm) | FOOD TECH RSA (FD Rm)                 | SCIENCE FOS (Sci Rm)  | ENGLISH COG (Eng Rm)                       | MATHS DT (Ma Rm)     | MATHS DT (Ma Rm)                | ENGLISH JE (Lib)     | ENGLISH COG (Eng Rm)            | SCIENCE FOS (Sci Rm) | ART RSA (Ar Rm)                      | MATHS DT (Ma Rm)     | ENGLISH COG (Eng Rm)                       | SCIENCE FOS (Sci Rm) | PE/Horticulture/Zumba RSA/FOS +TAs      |                       |                      |                      |
| P6                                   | 14.15 to 15.00 | YOGA FOS+ FY+TA Cabin                |                                |                       |                      |                                       | MATHS DT (Ma Rm)      | FS/PWL JE (IT Rm)                          |                      | Staff Training                  |                      |                                 |                      |                                      |                      | ENGLISH COG (Eng Rm)                       |                      | MATHS DT (Ma Rm)                        |                       |                      |                      |
| After School 15.00-16.00             |                | HOMEWORK CLUB                        |                                |                       |                      | HOMEWORK CLUB CODER DOJO CLUB         |                       |                                            |                      |                                 |                      |                                 |                      | HOMEWORK CLUB                        |                      |                                            |                      |                                         |                       |                      |                      |

## Sudoku 2

Can you fill this  
9×9 grid with  
numbers so  
that each row,  
column and  
3×3 section  
(marked in grey  
or white)  
contains all of  
the digits  
between 1 and  
9?

|   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|
| 2 |   | 9 | 1 |   | 4 | 5 |   | 7 |
| 8 |   |   |   | 7 | 9 | 2 | 1 |   |
| 6 |   | 7 | 3 |   | 2 | 4 | 8 |   |
| 9 |   | 3 | 8 |   | 5 | 7 |   | 1 |
| 4 |   |   | 2 | 6 |   |   | 3 |   |
| 1 | 8 |   |   | 9 |   | 6 |   |   |
| 7 |   | 1 |   | 2 |   | 3 |   | 6 |
|   | 9 |   | 4 |   |   | 1 |   |   |
| 3 |   | 6 | 9 |   | 7 |   | 5 | 4 |

What do you notice about the numbers in this square?

**But where did the 3s and the 7 in the bottom right hand corner come from ... ?**

## Making a Vedic Square

| 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 |
| 2 | 2 | 4 | 6 | 8 |
| 3 | 3 | 6 | 9 | 3 |
| 4 | 4 | 8 | 3 | 7 |

# Vedic Square

Can you fill in the missing numbers of this Vedic Square by multiplying the numbers in each column and row heading and then adding together any two digit number answers?

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 |   |   |   |   |   |
| 2 | 2 | 4 | 6 | 8 |   |   |   |   |   |
| 3 | 3 | 6 | 9 | 3 |   |   |   |   |   |
| 4 | 4 | 8 | 3 | 7 |   |   |   |   |   |
| 5 |   |   |   |   |   |   |   |   |   |
| 6 |   |   |   |   |   |   |   |   |   |
| 7 |   |   |   |   |   |   |   |   |   |
| 8 |   |   |   |   |   |   |   |   |   |
| 9 |   |   |   |   |   |   |   |   |   |

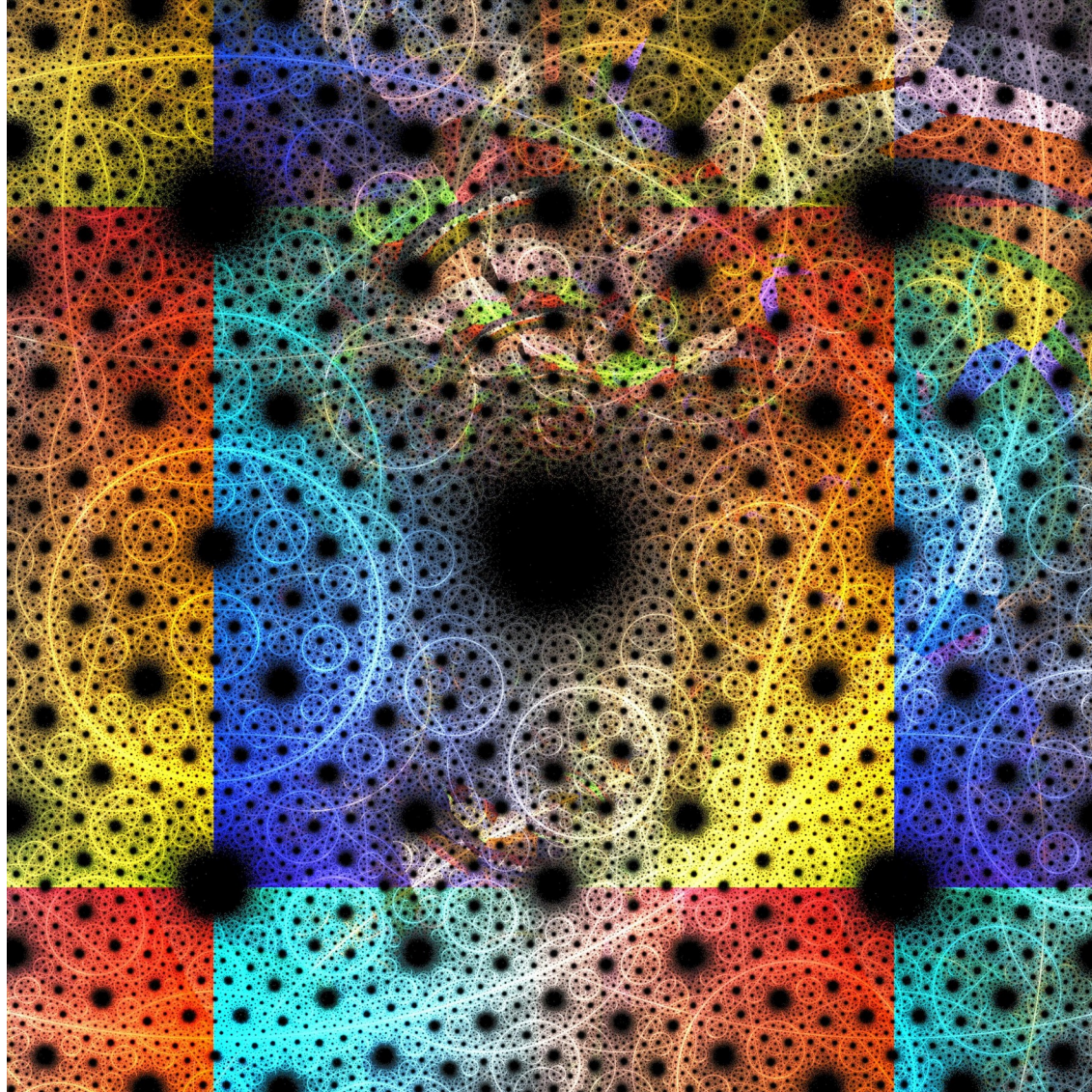
# Making patterns using the Vedic Square

Join all the 1s together using straight lines. Repeat this with other numbers using different coloured crayons. What patterns do you notice?

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 2 | 2 | 4 | 6 | 8 | 1 | 3 | 5 | 7 | 9 |
| 3 | 3 | 6 | 9 | 3 | 6 | 9 | 3 | 6 | 9 |
| 4 | 4 | 8 | 3 | 7 | 2 | 6 | 1 | 5 | 9 |
| 5 | 5 | 1 | 6 | 2 | 7 | 3 | 8 | 4 | 9 |
| 6 | 6 | 3 | 9 | 6 | 3 | 9 | 6 | 3 | 9 |
| 7 | 7 | 5 | 3 | 1 | 8 | 6 | 4 | 2 | 9 |
| 8 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 9 |
| 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 |

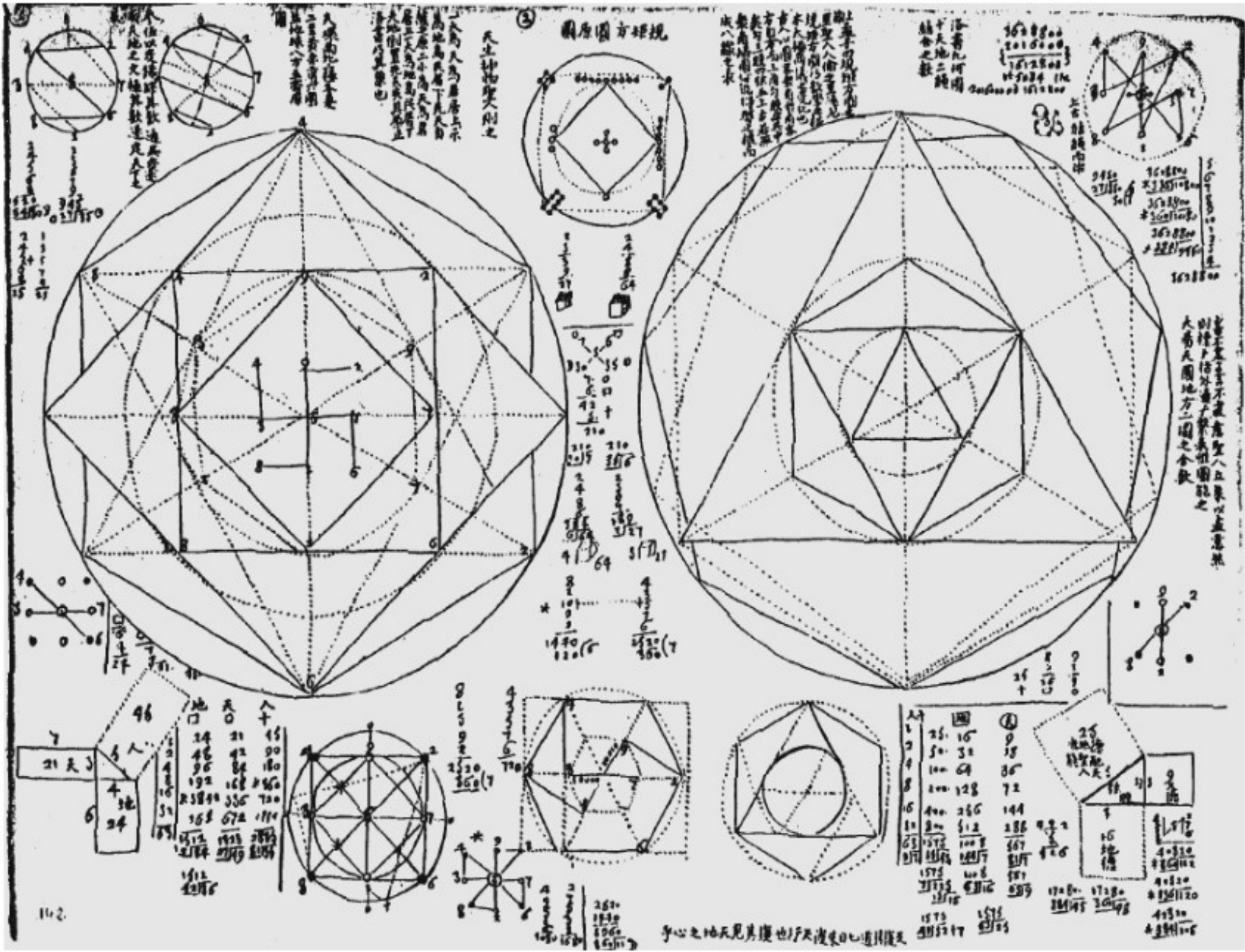
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 2 | 2 | 4 | 6 | 8 | 1 | 3 | 5 | 7 | 9 |
| 3 | 3 | 6 | 9 | 3 | 6 | 9 | 3 | 6 | 9 |
| 4 | 4 | 8 | 3 | 7 | 2 | 6 | 1 | 5 | 9 |
| 5 | 5 | 1 | 6 | 2 | 7 | 3 | 8 | 4 | 9 |
| 6 | 6 | 3 | 9 | 6 | 3 | 9 | 6 | 3 | 9 |
| 7 | 7 | 5 | 3 | 1 | 8 | 6 | 4 | 2 | 9 |
| 8 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 9 |
| 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 |

# The Magic of Magic Squares



For thousands of years people from different cultures across the world have been trying to understand patterns in nature, the seasons, the climate. Knowing patterns allows people to know when is best to plant or to prepare for winter. Even where to live and how to live. They used numbers, sometimes arranged in structures like Magic Squares, to try to help them. They carved them into temples, some even wore them round their necks. Even though they understood them in different ways they believed that they held power.

Here are some drawings by Jesuit Missionaries from Europe in China in 1668 who were trying to understand the Luo Shu Magic Square by joining up the numbers in different ways.



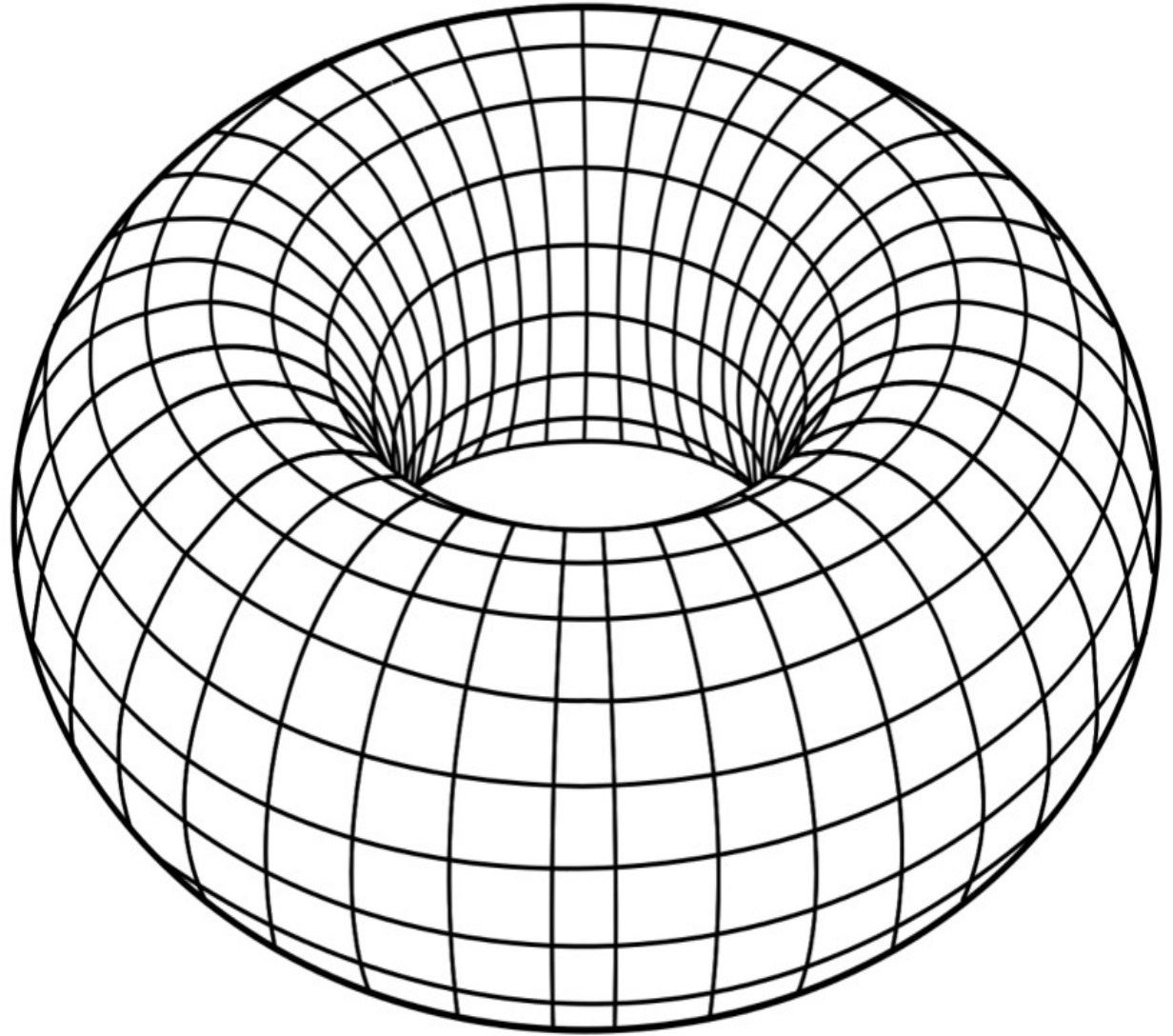
Further investigations with Magic Squares. This is a 9 x 9 Magic Square with its digital root numbers (as used in Vedic Squares) in the square below. What do you notice about the number patterns?

|    |    |    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|----|----|
| 47 | 58 | 69 | 80 | 1  | 12 | 23 | 34 | 45 |
| 57 | 68 | 79 | 9  | 11 | 22 | 33 | 44 | 46 |
| 67 | 78 | 8  | 10 | 21 | 32 | 43 | 54 | 56 |
| 77 | 7  | 18 | 20 | 31 | 42 | 53 | 55 | 66 |
| 6  | 17 | 19 | 30 | 41 | 52 | 63 | 65 | 76 |
| 16 | 27 | 29 | 40 | 51 | 62 | 64 | 75 | 5  |
| 26 | 28 | 39 | 50 | 61 | 72 | 74 | 4  | 15 |
| 36 | 38 | 49 | 60 | 71 | 73 | 3  | 14 | 25 |
| 37 | 48 | 59 | 70 | 81 | 2  | 13 | 24 | 35 |

How was the second square made from the first one?

|   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|
| 2 | 4 | 6 | 8 | 1 | 3 | 5 | 7 | 9 |
| 3 | 5 | 7 | 9 | 2 | 4 | 6 | 8 | 1 |
| 4 | 6 | 8 | 1 | 3 | 5 | 7 | 9 | 2 |
| 5 | 7 | 9 | 2 | 4 | 6 | 8 | 1 | 3 |
| 6 | 8 | 1 | 3 | 5 | 7 | 9 | 2 | 4 |
| 7 | 9 | 2 | 4 | 6 | 8 | 1 | 3 | 5 |
| 8 | 1 | 3 | 5 | 7 | 9 | 2 | 4 | 6 |
| 9 | 2 | 4 | 6 | 8 | 1 | 3 | 5 | 7 |
| 1 | 3 | 5 | 7 | 9 | 2 | 4 | 6 | 8 |

**When Magic Squares like this are folded they create a doughnut shape known as a 'Torus'. High quality electric transformers are made from these shapes. The 'harmonious' shape is very effective for making sure that electrical energy is not wasted.**



Certain numbers have been marked on this 27 x 27 Magic Square.

What pattern or shape can you see?

|     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 352 | 717 | 326 | 691 | 300 | 665 | 274 | 639 | 248 | 613 | 222 | 587 | 196 | 561 | 170 | 535 | 144 | 509 | 118 | 483 | 92  | 457 | 66  | 431 | 40  | 405 | 14  |
| 15  | 353 | 718 | 327 | 692 | 301 | 666 | 275 | 640 | 249 | 614 | 223 | 588 | 197 | 562 | 171 | 536 | 145 | 510 | 119 | 484 | 93  | 458 | 67  | 432 | 41  | 379 |
| 380 | 16  | 354 | 719 | 328 | 693 | 302 | 667 | 276 | 641 | 250 | 615 | 224 | 589 | 198 | 563 | 172 | 537 | 146 | 511 | 120 | 485 | 94  | 459 | 68  | 406 | 42  |
| 43  | 381 | 17  | 355 | 720 | 329 | 694 | 303 | 668 | 277 | 642 | 251 | 616 | 225 | 590 | 199 | 564 | 173 | 538 | 147 | 512 | 121 | 486 | 95  | 433 | 69  | 407 |
| 408 | 44  | 382 | 18  | 356 | 721 | 330 | 695 | 304 | 669 | 278 | 643 | 252 | 617 | 226 | 591 | 200 | 565 | 174 | 539 | 148 | 513 | 122 | 460 | 96  | 434 | 70  |
| 71  | 409 | 45  | 383 | 19  | 357 | 722 | 331 | 696 | 305 | 670 | 279 | 644 | 253 | 618 | 227 | 592 | 201 | 566 | 175 | 540 | 149 | 487 | 123 | 461 | 97  | 435 |
| 436 | 72  | 410 | 46  | 384 | 20  | 358 | 723 | 332 | 697 | 306 | 671 | 280 | 645 | 254 | 619 | 228 | 593 | 202 | 567 | 176 | 514 | 150 | 488 | 124 | 462 | 98  |
| 99  | 437 | 73  | 411 | 47  | 385 | 21  | 359 | 724 | 333 | 698 | 307 | 672 | 281 | 646 | 255 | 620 | 229 | 594 | 203 | 541 | 177 | 515 | 151 | 489 | 125 | 463 |
| 464 | 100 | 438 | 74  | 412 | 48  | 386 | 22  | 360 | 725 | 334 | 699 | 308 | 673 | 282 | 647 | 256 | 621 | 230 | 568 | 204 | 542 | 178 | 516 | 152 | 490 | 126 |
| 127 | 465 | 101 | 439 | 75  | 413 | 49  | 387 | 23  | 361 | 726 | 335 | 700 | 674 | 283 | 648 | 257 | 595 | 231 | 569 | 205 | 543 | 179 | 517 | 153 | 491 |     |
| 492 | 128 | 466 | 102 | 440 | 76  | 414 | 50  | 388 | 24  | 362 | 727 | 336 | 701 | 310 | 675 | 284 | 622 | 258 | 596 | 232 | 570 | 206 | 544 | 180 | 518 | 154 |
| 155 | 493 | 129 | 467 | 103 | 441 | 77  | 415 | 51  | 389 | 25  | 363 | 728 | 337 | 702 | 311 | 649 | 285 | 623 | 259 | 597 | 233 | 571 | 207 | 545 | 181 | 519 |
| 520 | 156 | 494 | 130 | 468 | 104 | 442 | 78  | 416 | 52  | 390 | 26  | 364 | 729 | 338 | 676 | 312 | 650 | 286 | 624 | 260 | 598 | 234 | 572 | 208 | 546 | 182 |
| 183 | 521 | 157 | 495 | 131 | 469 | 105 | 443 | 79  | 417 | 53  | 391 | 27  | 365 | 703 | 339 | 677 | 313 | 651 | 287 | 625 | 261 | 599 | 235 | 573 | 209 | 547 |
| 548 | 184 | 522 | 158 | 496 | 132 | 470 | 106 | 444 | 80  | 418 | 54  | 392 | 1   | 366 | 704 | 340 | 678 | 314 | 652 | 288 | 626 | 262 | 600 | 236 | 574 | 210 |
| 211 | 549 | 185 | 523 | 159 | 497 | 133 | 471 | 107 | 445 | 81  | 419 | 28  | 393 | 2   | 367 | 705 | 341 | 679 | 315 | 653 | 289 | 627 | 263 | 601 | 237 | 575 |
| 576 | 212 | 550 | 186 | 524 | 160 | 498 | 134 | 472 | 108 | 446 | 55  | 420 | 29  | 394 | 3   | 368 | 706 | 342 | 680 | 316 | 654 | 290 | 628 | 264 | 602 | 238 |
| 239 | 577 | 213 | 551 | 187 | 525 | 161 | 499 | 135 | 473 | 82  | 447 | 56  | 421 | 30  | 395 | 4   | 369 | 707 | 343 | 681 | 317 | 655 | 291 | 629 | 265 | 603 |
| 604 | 240 | 578 | 214 | 552 | 188 | 526 | 162 | 500 | 109 | 474 | 83  | 448 | 57  | 422 | 31  | 396 | 5   | 370 | 708 | 344 | 682 | 318 | 656 | 292 | 630 | 266 |
| 267 | 605 | 241 | 579 | 215 | 553 | 189 | 527 | 136 | 501 | 110 | 475 | 84  | 449 | 58  | 423 | 32  | 397 | 6   | 371 | 709 | 345 | 683 | 319 | 657 | 293 | 631 |
| 632 | 268 | 606 | 242 | 580 | 216 | 554 | 163 | 528 | 137 | 502 | 111 | 476 | 85  | 450 | 59  | 424 | 33  | 398 | 7   | 372 | 710 | 346 | 684 | 320 | 658 | 294 |
| 295 | 633 | 269 | 607 | 243 | 581 | 190 | 555 | 164 | 529 | 138 | 503 | 112 | 477 | 86  | 451 | 60  | 425 | 34  | 399 | 8   | 373 | 711 | 347 | 685 | 321 | 659 |
| 660 | 296 | 634 | 270 | 608 | 217 | 582 | 191 | 556 | 165 | 530 | 139 | 504 | 113 | 478 | 87  | 452 | 61  | 426 | 35  | 400 | 9   | 374 | 712 | 348 | 686 | 322 |
| 323 | 661 | 297 | 635 | 244 | 609 | 218 | 583 | 192 | 557 | 166 | 531 | 140 | 505 | 114 | 479 | 88  | 453 | 62  | 427 | 36  | 401 | 10  | 375 | 713 | 349 | 687 |
| 688 | 324 | 662 | 271 | 636 | 245 | 610 | 219 | 584 | 193 | 558 | 167 | 532 | 141 | 506 | 115 | 480 | 89  | 454 | 63  | 428 | 37  | 402 | 11  | 376 | 714 | 350 |
| 351 | 689 | 298 | 663 | 272 | 637 | 246 | 611 | 220 | 585 | 194 | 559 | 168 | 533 | 142 | 507 | 116 | 481 | 90  | 455 | 64  | 429 | 38  | 403 | 12  | 377 | 715 |
| 716 | 325 | 690 | 299 | 664 | 273 | 638 | 247 | 612 | 221 | 586 | 195 | 560 | 169 | 534 | 143 | 508 | 117 | 482 | 91  | 456 | 65  | 430 | 39  | 404 | 13  | 378 |

| What do you notice about these numbers?                                    | <b>C</b> | <b>G</b> | <b>D</b> | <b>A</b> | <b>E</b> | <b>B</b> | <b>F#</b> | <b>Db</b> |
|----------------------------------------------------------------------------|----------|----------|----------|----------|----------|----------|-----------|-----------|
| What do you think they represent?                                          | 1        | 3        | 9        | 27       | 81       | 243      | 729       | 2187      |
|                                                                            | 2        | 6        | 18       | 54       | 162      | 486      | 1458      | 4374      |
|                                                                            | 4        | 12       | 36       | 108      | 324      | 972      | 2916      | 8748      |
|                                                                            | 8        | 24       | 72       | 216      | 648      | 1944     | 5832      | 17496     |
| It is these numbers were marked on the magic square in the previous slide. | 16       | 48       | 144      | 432      | 1296     | 3888     | 11664     | 34992     |
|                                                                            | 32       | 96       | 288      | 864      | 2592     | 7776     | 23328     | 69984     |
|                                                                            | 64       | 192      | 576      | 1728     | 5184     | 15552    | 46656     | 139968    |
|                                                                            | 128      | 384      | 1152     | 3456     | 10368    | 31104    | 93312     | 279936    |
|                                                                            | 256      | 768      | 2304     | 6912     | 20736    | 62208    | 186624    |           |
|                                                                            | 512      | 1536     | 4608     | 13824    | 41472    | 124416   |           |           |

In 527 CE  
Varahamihira from  
India described a  
magic square for  
making perfumes.

Each cell in the  
square represents  
a different  
ingredient and  
each number gives  
the proportion of  
the ingredient.

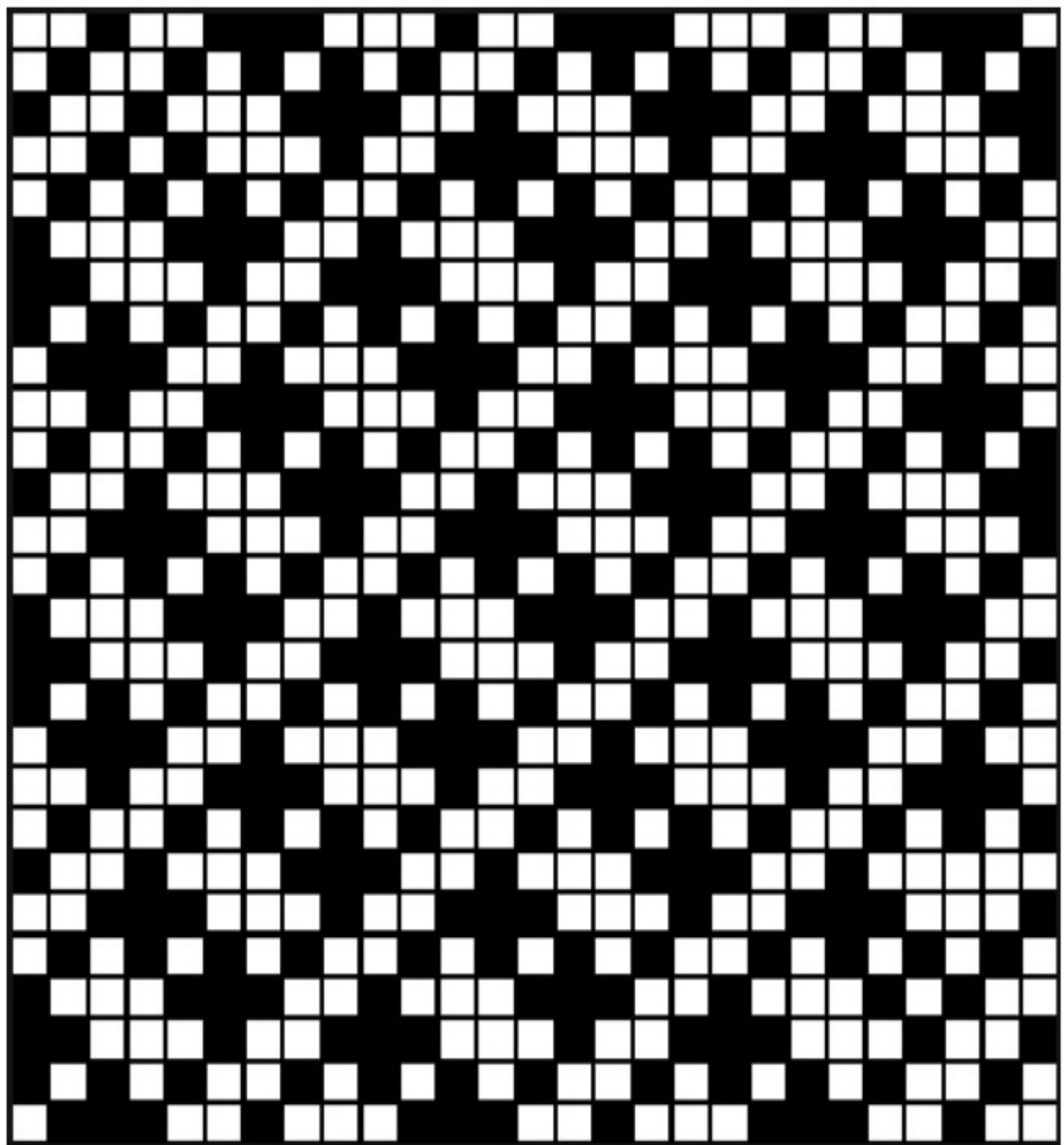
A different perfume  
is created by  
adding the given  
volume of each of  
the four ingredients  
together along  
each row, column  
or diagonal.

What will be the  
volume of each  
perfume?

|   |   |   |   |
|---|---|---|---|
| 2 | 3 | 5 | 8 |
| 5 | 8 | 2 | 3 |
| 4 | 1 | 7 | 6 |
| 7 | 6 | 4 | 1 |



Here is a  $27 \times 27$  Magic Square with the even numbered squares shaded black. What patterns do you notice?



This pattern was created on this Magic Square by starting at the top left and colouring in the cells of the numbers that are in their correct sequence in yellow (i.e. 1, 4, 5 etc.). The cells with numbers that go in the opposite direction (from bottom right to top left) are coloured in purple.

|    |    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|----|
| 1  | 63 | 62 | 4  | 5  | 59 | 58 | 8  |
| 56 | 10 | 11 | 53 | 52 | 14 | 15 | 49 |
| 48 | 18 | 19 | 45 | 44 | 22 | 23 | 41 |
| 25 | 39 | 38 | 28 | 29 | 35 | 34 | 32 |
| 33 | 31 | 30 | 36 | 37 | 27 | 26 | 40 |
| 24 | 42 | 43 | 21 | 20 | 46 | 47 | 17 |
| 16 | 50 | 51 | 13 | 12 | 54 | 55 | 9  |
| 57 | 7  | 6  | 60 | 61 | 3  | 2  | 64 |

|    |    |    |    |
|----|----|----|----|
| 1  | 15 | 14 | 4  |
| 12 | 6  | 7  | 9  |
| 8  | 10 | 11 | 5  |
| 13 | 3  | 2  | 16 |

|    |    |    |    |
|----|----|----|----|
| 16 | 2  | 3  | 13 |
| 5  | 11 | 10 | 8  |
| 9  | 7  | 6  | 12 |
| 4  | 14 | 15 | 1  |

Try it with one of the two

Now try  
doing the  
same thing  
with this 8  
x 8 Magic  
Square

|    |    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|----|
| 1  | 63 | 3  | 61 | 60 | 6  | 58 | 8  |
| 56 | 55 | 11 | 12 | 13 | 14 | 50 | 49 |
| 17 | 18 | 46 | 45 | 44 | 43 | 23 | 24 |
| 40 | 26 | 38 | 28 | 29 | 35 | 31 | 33 |
| 32 | 34 | 30 | 36 | 37 | 27 | 39 | 25 |
| 41 | 42 | 22 | 21 | 20 | 19 | 47 | 48 |
| 16 | 15 | 51 | 52 | 53 | 54 | 10 | 9  |
| 57 | 7  | 59 | 5  | 4  | 62 | 2  | 64 |

Now try doing the same thing with one of these  $6 \times 6$  Magic Squares. This time you will need four different colours and to start looking for numbers going up in sequence starting from each one of the four corner squares.

|    |    |    |    |    |    |
|----|----|----|----|----|----|
| 6  | 32 | 3  | 34 | 35 | 1  |
| 7  | 11 | 27 | 28 | 8  | 30 |
| 19 | 14 | 16 | 15 | 23 | 24 |
| 18 | 20 | 22 | 21 | 17 | 13 |
| 25 | 29 | 10 | 9  | 26 | 12 |
| 36 | 5  | 33 | 4  | 2  | 31 |

|    |    |    |    |    |    |
|----|----|----|----|----|----|
| 1  | 5  | 33 | 34 | 32 | 6  |
| 30 | 8  | 28 | 9  | 11 | 25 |
| 18 | 23 | 15 | 16 | 20 | 19 |
| 24 | 14 | 21 | 22 | 17 | 13 |
| 7  | 26 | 10 | 27 | 29 | 12 |
| 31 | 35 | 4  | 3  | 2  | 36 |

Other patterns can be seen when the number in the centre of a Magic Square (with an odd number of rows and columns) is reduced to zero (just as was done with the Luo Shu Magic Square in slide 6).

|    |     |     |    |     |
|----|-----|-----|----|-----|
| -9 | 12  | 5   | -2 | -6  |
| 1  | 7   | -11 | 4  | -1  |
| -8 | -3  | 0   | 3  | 8   |
| 10 | -4  | 11  | -7 | -10 |
| 6  | -12 | -5  | 2  | 9   |

What do you notice?

The pattern is a little different if similar opposite pairs of numbers are created with a Magic Square with an even number of rows and columns.

What do you notice?

|     |     |     |    |
|-----|-----|-----|----|
| 1   | -3  | 11  | -9 |
| -5  | 7   | -15 | 13 |
| -11 | 9   | -1  | 3  |
| 15  | -13 | 5   | -7 |

Watch this short film clip on resonance  
patterns

<https://www.youtube.com/watch?v=hlgmiDnmVdU>



Can you find any similarities between any of the Magic Square patterns and the metal plate resonance patterns?

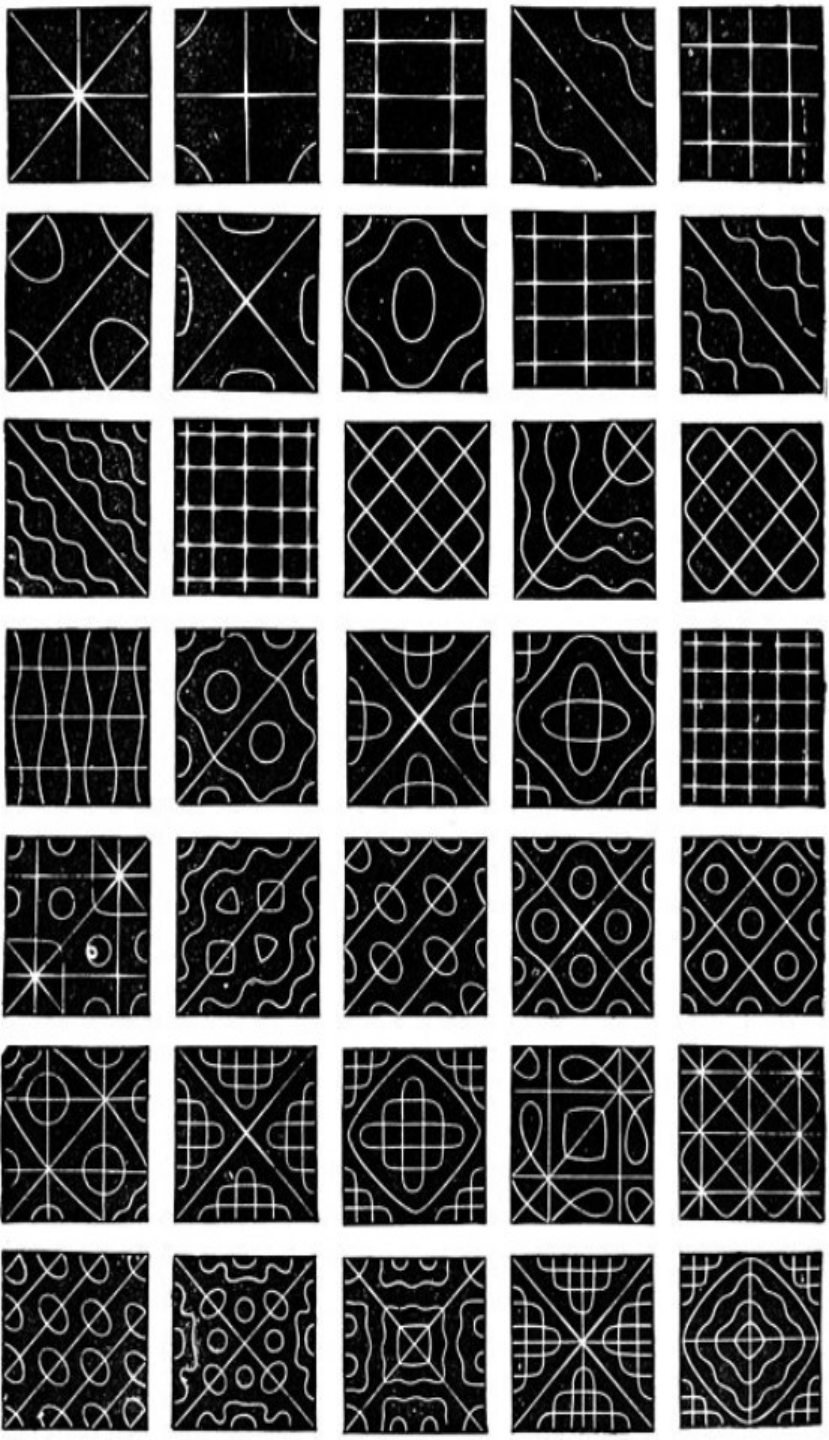
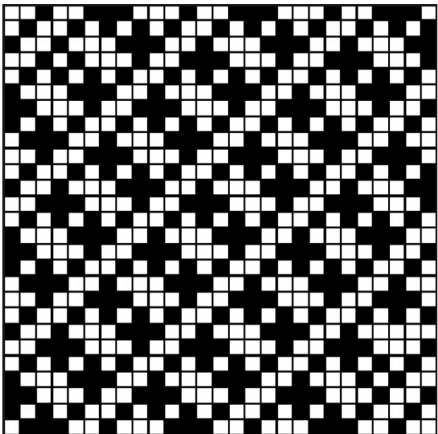
|    |    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|----|
| 1  | 63 | 62 | 4  | 5  | 59 | 58 | 8  |
| 56 | 10 | 11 | 53 | 52 | 14 | 15 | 49 |
| 48 | 18 | 19 | 45 | 44 | 22 | 23 | 41 |
| 25 | 39 | 38 | 28 | 29 | 35 | 34 | 32 |
| 33 | 31 | 30 | 36 | 37 | 27 | 26 | 40 |
| 24 | 42 | 43 | 21 | 20 | 46 | 47 | 17 |
| 16 | 50 | 51 | 13 | 12 | 54 | 55 | 9  |
| 57 | 7  | 6  | 60 | 61 | 3  | 2  | 64 |

|    |    |    |    |    |    |
|----|----|----|----|----|----|
| 6  | 32 | 3  | 34 | 35 | 1  |
| 7  | 11 | 27 | 28 | 8  | 30 |
| 19 | 14 | 16 | 15 | 23 | 24 |
| 18 | 20 | 22 | 21 | 17 | 13 |
| 25 | 29 | 10 | 9  | 26 | 12 |
| 36 | 5  | 33 | 4  | 2  | 31 |

|    |    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|----|
| 1  | 63 | 3  | 61 | 60 | 6  | 58 | 8  |
| 56 | 55 | 11 | 12 | 13 | 14 | 50 | 49 |
| 17 | 18 | 46 | 45 | 44 | 43 | 23 | 24 |
| 40 | 26 | 38 | 28 | 29 | 35 | 31 | 33 |
| 32 | 34 | 30 | 36 | 37 | 27 | 39 | 25 |
| 41 | 42 | 22 | 21 | 20 | 19 | 47 | 48 |
| 16 | 15 | 51 | 52 | 53 | 54 | 10 | 9  |
| 57 | 7  | 59 | 5  | 4  | 62 | 2  | 64 |

|    |     |     |    |     |
|----|-----|-----|----|-----|
| -9 | 12  | 5   | -2 | -6  |
| 1  | 7   | -11 | 4  | -1  |
| -8 | -3  | 0   | 3  | 8   |
| 10 | -4  | 11  | -7 | -10 |
| 6  | -12 | -5  | 2  | 9   |

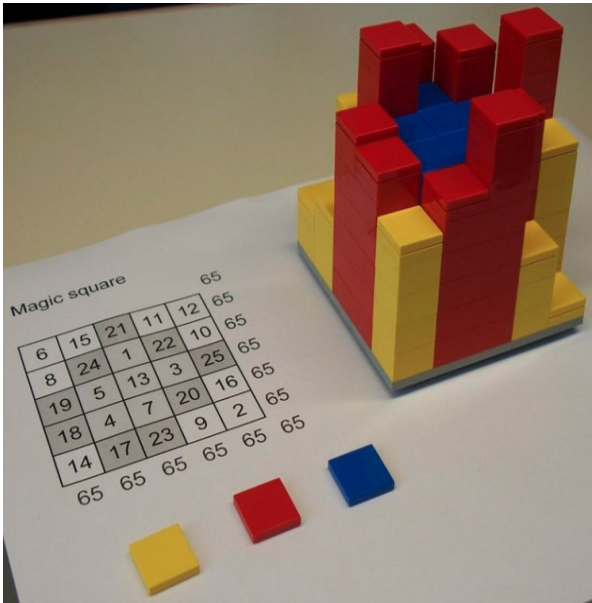
|     |     |     |    |
|-----|-----|-----|----|
| 1   | -3  | 11  | -9 |
| -5  | 7   | -15 | 13 |
| -11 | 9   | -1  | 3  |
| 15  | -13 | 5   | -7 |



If a Magic Square is made of Lego blocks with higher blocks for larger numbers and water is tipped on it what do you think will happen?

Some Magic Squares, like the one in the second photo will hold a lot of water.

If the Magic Square has been made like the Luo Shu square, only larger, it will contain the most number of ponds. It will lead to a greater spread of water. Perhaps if it were a landscape it would lead to less serious flooding, like in the message in the turtle story at the beginning?



|     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 137 | 282 | 121 | 266 | 105 | 250 | 89  | 234 | 73  | 218 | 57  | 202 | 41  | 186 | 25  | 170 | 9   |
| 10  | 138 | 283 | 122 | 267 | 106 | 251 | 90  | 235 | 74  | 219 | 58  | 203 | 42  | 187 | 26  | 154 |
| 155 | 11  | 139 | 284 | 123 | 268 | 107 | 252 | 91  | 236 | 75  | 220 | 59  | 204 | 43  | 171 | 27  |
| 28  | 156 | 12  | 140 | 285 | 124 | 269 | 108 | 253 | 92  | 237 | 76  | 221 | 60  | 188 | 44  | 172 |
| 173 | 29  | 157 | 13  | 141 | 286 | 125 | 270 | 109 | 254 | 93  | 238 | 77  | 205 | 61  | 189 | 45  |
| 46  | 174 | 30  | 158 | 14  | 142 | 287 | 126 | 271 | 110 | 255 | 94  | 222 | 78  | 206 | 62  | 190 |
| 191 | 47  | 175 | 31  | 159 | 15  | 143 | 288 | 127 | 272 | 111 | 239 | 95  | 223 | 79  | 207 | 63  |
| 64  | 192 | 48  | 176 | 32  | 160 | 16  | 144 | 289 | 128 | 256 | 112 | 240 | 96  | 224 | 80  | 208 |
| 209 | 65  | 193 | 49  | 177 | 33  | 161 | 17  | 145 | 273 | 129 | 257 | 113 | 241 | 97  | 225 | 81  |
| 82  | 210 | 66  | 194 | 50  | 178 | 34  | 162 | 1   | 146 | 274 | 130 | 258 | 114 | 242 | 98  | 226 |
| 227 | 83  | 211 | 67  | 195 | 51  | 179 | 18  | 163 | 2   | 147 | 275 | 131 | 259 | 115 | 243 | 99  |
| 100 | 228 | 84  | 212 | 68  | 196 | 35  | 180 | 19  | 164 | 3   | 148 | 276 | 132 | 260 | 116 | 244 |
| 245 | 101 | 229 | 85  | 213 | 52  | 197 | 36  | 181 | 20  | 165 | 4   | 149 | 277 | 133 | 261 | 117 |
| 118 | 246 | 102 | 230 | 69  | 214 | 53  | 198 | 37  | 182 | 21  | 166 | 5   | 150 | 278 | 134 | 262 |
| 263 | 119 | 247 | 86  | 231 | 70  | 215 | 54  | 199 | 38  | 183 | 22  | 167 | 6   | 151 | 279 | 135 |
| 136 | 264 | 103 | 248 | 87  | 232 | 71  | 216 | 55  | 200 | 39  | 184 | 23  | 168 | 7   | 152 | 280 |
| 281 | 120 | 265 | 104 | 249 | 88  | 233 | 72  | 217 | 56  | 201 | 40  | 185 | 24  | 169 | 8   | 153 |

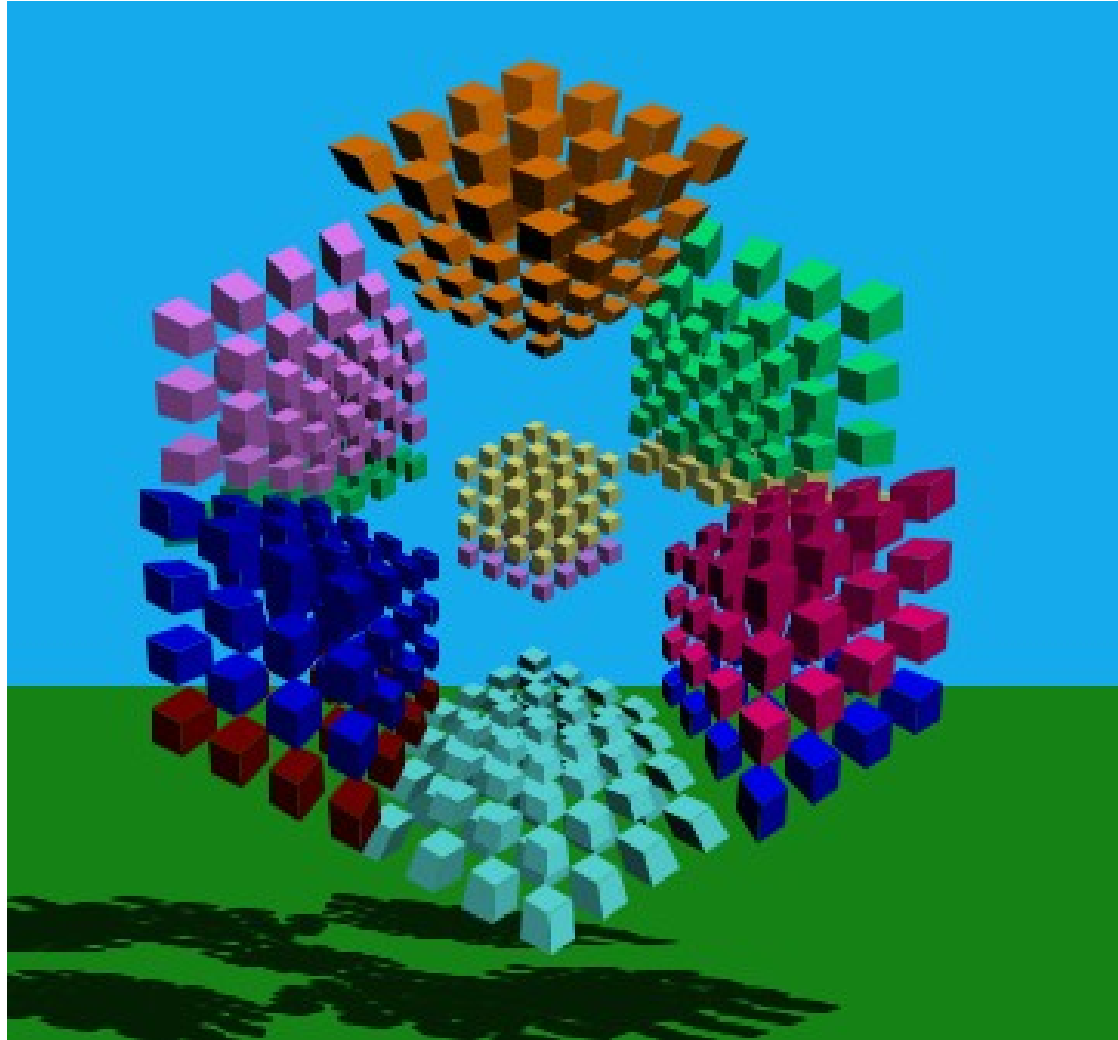
Film clip P4C stimulus:

<https://www.youtube.com/watch?v=Y8SA0gtSBNs>

People are still trying to understand Magic Squares. This film shows some patterns created from the Luo Shu Magic Square using 3D graphics.

As you watch it, think about what you have been learning about Magic Squares and try to come up with some philosophical questions.

# Extending The Learning



A Geomagic Square based on Luo Shu. There are solutions to the rows on the right.  
Can you solve the columns or diagonals?

